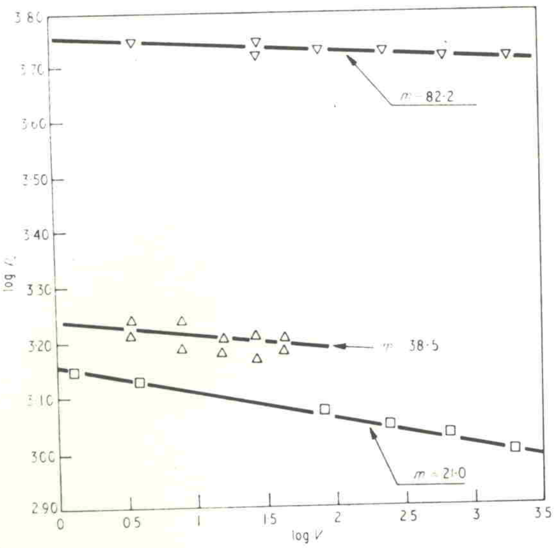

□ Steel A.
△ Steel B.
▽ Steel C.

Fig. 1. Variation of p_e as a function of cylinder size

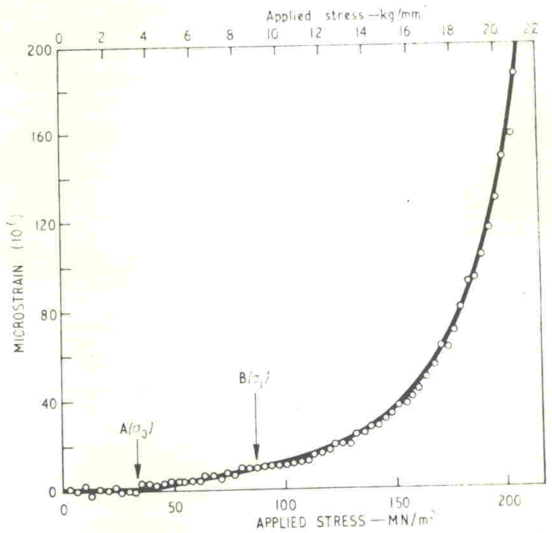

Fig. 2. Microplasticity curve for steel A at +20°C

defects are naturally more numerous in a large part, and their presence accounts for the existence of a size effect as regards fracture. Weibull (5) has evolved a statistical theory of the strength of materials which is based on this explanation.

It appears reasonable to assume that Weibull's statistical theory also holds in the case of plastic strain. Richards has used this concept to explain the size effect relating to the upper yield stress of mild steel, both in tension (6) and in bending (7). In the case of plastic strain, the microscopic defects in the material would no longer be microcracks but very small regions in which plastic strain is very easy—for instance, small regions containing dislocation sources.

Assuming that the statistical theory put forward by Weibull applies to plastic strain in thick-walled cylinders, the pressure, p_e, corresponding to the onset of plastic strain, must decrease as the size increases according to the law:

$$p_e = \frac{p_{e1}}{V^{1/m}} \quad . \quad . \quad . \quad . \quad (1)$$

where p_{e1} is the pressure for the onset of plastic strain in a cylinder of unit volume, V is the volume of the cylinder considered, and m is a constant for a given material. In Fig. 1, in which $\log p_e$ has been plotted versus $\log V$, the volume of the smallest cylinder has been selected as the unit volume. This figure shows that a size effect actually exists, and that the experimental results fit in satisfactorily with Weibull's theory. It is seen that the exponent m increases with increasing yield stress of the material, which means that the size effect is sharper in a mild steel than it is in a high-strength steel.

CALCULATION OF THE STRESS DISTRIBUTION IN A THICK-WALLED CYLINDER

In order to understand better the phenomena which precede the onset of plastic strain in a thick-walled cylinder, and to be in a position to calculate the complete stress distribution in such a cylinder, the plastic properties of the material must be known as fully as possible. In this respect, three important aspects have to be discussed: (1) the microplasticity of the steel, (2) the criterion of yielding, and (3) the general relation between stress and plastic strain.

Microplasticity of the steel

When discussing the mechanical behaviour of a steel in terms of the conventional tensile test, it is generally assumed that the upper yield stress constitutes a natural limit below which the metal strains elastically, and above which it strains plastically. A detailed investigation of the phenomenon shows that the conventional tensile test does not provide a full description of the mechanical properties. In actual fact there is a transition between the elastic and plastic fields which can be studied more thoroughly through microplasticity measurements.

In order to determine the microplasticity curve of a material, use is made of a deadweight tensile testing machine in which the applied stress is increased according to a linear law ($d\sigma/dt$ = constant). The test is carried out at a carefully controlled temperature (±0·02 degC), and the elongations are measured by means of a high-sensitivity extensometer ($\pm 1 \times 10^{-6}$) which is photographed at regular time intervals during the test (8).

Fig. 2 shows the microplasticity curve of steel A. This curve was recorded at a temperature of +20°C, using a loading rate of $7{\cdot}6 \times 10^{-3}$ kg/mm² s. At the start of the test, i.e. below point A corresponding to a given threshold value, σ_0, no microstrains are observed, and the strains are thus purely elastic. At this moment the dislocation density is of the order of 10^8 cm/cm³; in other words, the dislocations are too few to give rise to a measurable plastic strain. Between A and B, where B corresponds to a second threshold value, σ_1, microstrains are observed which increase according to a linear law. They can be attributed to small movements of the dislocations existing in the steel, without any formation of additional dislocations (9). At point B, the dislocation sources come into play, and progressively increase the dislocation density up to a value of the order of 10^{10}–10^{11} cm/cm³, at which level macroscopic plastic strain sets in. This part of the test is characterized by a parabolic law of microstraining: